２０１５年度

慶應義塾大学大学院入試問題

経済学研究科（修士課程）

２０１４年9月１１日 実施

<table>
<tr><td>科
目
名</td><td>Economics
(English)</td><td>受
験
番
号</td><td></td><td>氏
名</td><td></td></tr>
</table>

注意事項（Please note:）

1. This set of problems contains 8 pages (including the cover page).
2. There are seven problems from which you should choose two to answer. Each problem should be answered on a separate answer sheet. Please write the number of the problem you are answering on each answer sheet.
3. If you answer two or more problems on one answer sheet, only the first answer will be treated as a valid answer. Everything after the first answer will not be marked.
4. Answer in English.
5. Although the problem sheets will not be collected after the examination, please write your name and exam registration number (受験番号, jyuken-bango) on the cover page.

Problem 1. Answer both (1) and (2).

(1) Consider a pure-exchange economy in which two consumers, 1 and 2, trade two goods, X and Y. Consumer 1's utility function is $u_1(x_1, y_1) = x_1 \cdot y_1$ and consumer 2's is $u_2(x_2, y_2) = (x_2)^2 \cdot y_2$. Consumer 1 has 2 units of good X and 4 units of good Y, which she sells at market prices to earn her income. Consumer 2 has 6 units of good X and 3 units of good Y to get his income. Below, normalize the price of good X as 1.

(i) Let p be the price of good Y. Compute the demand for good X of each consumer.

(ii) Compute the competitive price of good Y and the competitive allocation of the goods among the two consumers.

(2) A piano shop owner is thinking to hire a salesperson among three candidates: Ms. A, Ms. B, and Ms. C. The owner will offer a piecemeal-pay contract, that is, to pay w if a piano is sold but nothing if a piano is not sold. (The owner will hire only one salesperson for a period of time, in which at most one piano can be sold. All agents are risk neutral.)

Ms. A can sell a piano with probability 0.5 and her disutility of labor (in monetary units) is 5. Hence, if she is offered a piecemeal-pay contract which pays w if and only if a piano is sold, she accepts the contract as long as $u_A(w) = (0.5)w - 5 \geqq 0$ is satisfied.

Ms. B can sell a piano with probability 2/3 and her disutility of labor is 8. Ms. C can sell a piano with probability 1/5 and her disutility of labor is 1.

(i) Who accepts the cheapest w? Explain your answer.

(ii) Let $R(\geqq 0)$ be the piano shop's revenue from a piano sale. The owner wants to maximize $s \cdot (R - w)$, where s is the probability of a piano sale and w is the piecemeal-pay. As a function of R, specify which candidate should be hired. Your answer should look like, for example, "if $0 \leqq R \leqq 5$, hire Ms. Z, and if $5 < R$, hire Ms. W". You can assume that the owner offers the minimum w that a candidate accepts.

(iii) Write the economic implications of your answer to (ii), using the word "trade-off".

Problem 2.

Consider a 3-period optimal growth model. The time runs from period 0 to period 2. The representative agent solves the next problem:

$$\max_{c_0, c_1, c_2, x_1, x_2} \quad u(c_0) + \beta u(c_1) + \beta^2 u(c_2)$$

subject to

$$\begin{aligned} &x_0 \text{ is given} \\ &c_0 + x_1 = f(x_0) \\ &c_1 + x_2 = f(x_1) \\ &c_2 = f(x_2) \end{aligned}$$

Here, c_t $(t = 0, 1, 2)$ denotes the t-th period's consumption and x_t $(t = 0, 1, 2)$ denotes the capital stock at the beginning of the t-th period, all of which are nonnegative real numbers. In particular, x_0 is a positive real number. The utility index $u : \mathbb{R}_+ \to \mathbb{R}$ is a differentiable, increasing and concave function and the production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is a differentiable, increasing and concave function which satisfies $f(0) = 0$. (Therefore, x_t $(t = 1, 2)$ is an investment made in the $(t-1)$-th period and it completely depreciates by the production process.) Also, β is a constant such that $0 < \beta < 1$.

(1) Derive the relationship between the variables optimally chosen in the 0-th period and the 1-st period (the so-called *Euler equation*). Fully justify your derivation.

(2) Why must the Euler equation found in (1) hold? Explain from the viewpoint of economics.

(3) Specify the utility index u by $u(c) = \ln(c)$ and the production function f by $f(x) = x^\alpha$, where α is a constant such that $0 < \alpha < 1$. Then, find the optimal investment in period 1 (x_2) as a function of x_1.

(4) Under the specifications made in (3), find the optimal investment in period 0 (x_1) as a function of x_0.

Problem 3

Explain what is the Class-Exploitation Correspondence Principle (CECP) which was initiated by Analytical Marxists. Furthermore, explain what is the definition of 'class' in this theory, by comparing it with other definitions of 'class' in Marxian economics.

Problem 4.

Suppose that we have n pairs of observations (Y_1, X_1), …, (Y_n, X_n) and the relationship among them is determined by the following regression model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \quad i = 1, \ldots, n,$$

where $\varepsilon_1, \ldots, \varepsilon_n$ are unobservable, mutually independently and identically distributed random variables with $E(\varepsilon_i) = 0$ and $\text{var}(\varepsilon_i) = \sigma^2$. Unknown parameters are α, β, and σ^2 in this model.

(a) Calculate the ordinary least square (OLS) estimators for α and β, which are denoted by $\hat{\alpha}$ and $\hat{\beta}$, respectively.

(b) Prove that $\hat{\alpha}$ and $\hat{\beta}$ are unbiased estimators.

(c) Calculate the variances of $\hat{\alpha}$ and $\hat{\beta}$.

(d) Calculate the covariance between $\hat{\alpha}$ and $\hat{\beta}$.

(e) Define the coefficient of determination, R^2, as $R^2 = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2$, where $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$. Show that the following holds. $R^2 = 1 - \left(\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \right)$.

(f) Let X_1, …, X_n be iid samples with $E(X_i) = \mu$ and $\text{var}(X_i) = \sigma^2$. Show that the sample variance $s^2 = (n-1)^{-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$; where $\bar{X} = n^{-1} \sum_{i=1}^{n} X_i$, is an unbiased estimator for σ^2.

(g) A random variable X satisfies that $E(|X - b|^2) < \infty$ for some constant b. Prove Chebyshev's inequality: $\Pr(|X - b| \geq \varepsilon) \leq E(|X - b|^2)/\varepsilon^2$, where ε is a constant such that $\varepsilon > 0$.

(h) Suppose that $E(X_i^2) < \infty$ for a sequence of random variables X_i, $i = 1, \ldots, n$. When X_n converges in mean square (or quadratic mean) to a constant b, show that X_n also converges in probability to b using Chebyshev's inequality. Also write explicitly the definitions of "convergence in mean square (or quadratic mean)" and "convergence in probability".

Problem 5

Answer one of the following questions.

A. Name two conventional indicators most frequently used for measuring income distributions, and discuss the problem of "value judgment" involved.

B. Explain the effects of rent control on the quantity and rent of rental housing, using separate diagrams for short-term and long-term effects. Also, explain why rent control is inefficient as a policy for helping the poor.

C. Explain taxation principles which are desirable for local taxes. In Japan, the share of property taxes in local tax revenues is higher than that in the national tax revenue. Discuss which tax principle this fact accords with.

Problem 6

Suppose you are writing a textbook on the History of Economics from the years 1500 to 2000, comprising 4 or 5 chapters excluding the Introduction and Conclusion. List the title of each chapter, and briefly sketch the contents of any 2 chapters (in less than 400 words for each chapter).

Problem 7.

Choose any region or country and discuss the role played by its currency system and financial system, in its industrialization (or economic development). Make sure to use historical facts and discuss from the perspective of economic history.